Supermacro Photography and Illuminance

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There are three basic tools for capturing greater than life-size images with a 1:1 macro lens --- extension tubes, teleconverters, and supplementary close-up lenses. Close-up lenses increase the light intensity at the image plane, while extension tubes and teleconverters decrease it. These effects are explained below using elementary optics theory for single-element thin lenses. The conclusions are qualitatively applicable to all types of lenses.

First, some definitions. Roughly speaking, **luminance** and **illuminance** are the light energy per unit area per unit time being emitted by or incident on an object, respectively. For precise definitions see the article entitled “Luminance, Illuminance, Lumens, Lux, . . .”. In each of the following sections, the luminance distribution of the object being photographed is assumed to always be the same. The purpose of the analyses is to determine what happens to the illuminance at the **image capture plane** under various supermacro scenarios. In the context of digital photography, the image capture plane is the plane of the image sensor.

Close-up Lenses

Figure 1 shows a thin “primary” converging lens of focal length “f”, an object at distance “o” from the center of the lens, and its image at a distance “i”.

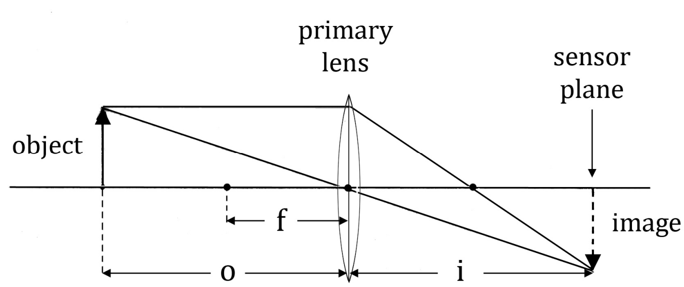

Figure 1

These quantities are related by the thin-lens formula

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \tag{1}$$

The ratio of the image height to the object height, i.e. the magnification, is given by the expression

$$m = \frac{f}{o - f} = \frac{i - f}{f} \tag{2}$$

Assume now that the distance "i" is equal to the maximum allowable separation between the lens and the image sensor. Then "o" is the minimum working distance of the primary lens, i.e. the "closest focusing distance", and "m" is the maximum achievable magnification of the primary lens. A magnification ratio of 1 corresponds to the case where o = i = 2f.

Figure 2 shows the same primary lens coupled with a thin close-up lens of focal length f_d positioned a distance "x" in front of the primary lens. The maximum working distance for this lens combination is f_d , while the minimum working distance, o_d , is the one that produces a virtual image at the distance "o" from the primary lens, as shown. The virtual image is therefore at the minimum working distance of the primary lens.

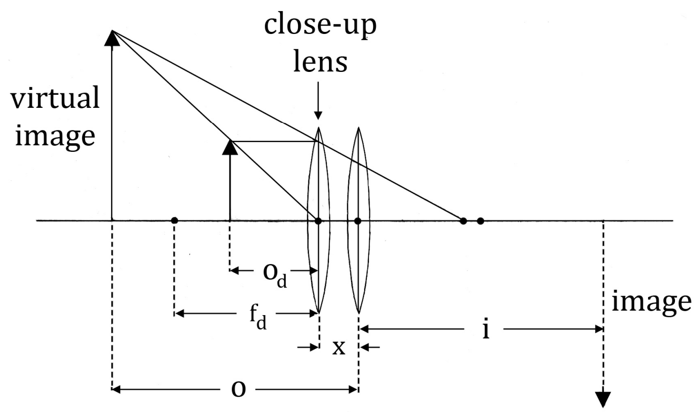

Figure 2

For this configuration, the thin-lens formula yields

$$\frac{1}{o_d} - \frac{1}{o - x} = \frac{1}{f_d} \tag{3}$$

By analogy with Equation 2, the height of the close-up lens' virtual image relative to the height of the object is

$$M = \frac{f_d}{f_d - o_d} = \frac{f_d + o - x}{f_d} \tag{4}$$

This virtual image, which is taller than the object by the factor M, is now the object for the primary lens. For this lens combination, the net magnification of the image at the sensor, m_d , will be greater than the magnification of the primary lens alone by the factor M, i.e.

$$\frac{m_d}{m} = M \tag{5}$$

Note that an increase in magnification by this ratio implies that both the height and width of the image at the sensor increase by this factor, so that the area increases by the square of this factor. An image that now completely covers the sensor area used to occupy only a fraction of it, the fraction being $(m/m_d)^2$.

From Equation 3 the ratio of the minimum working distances is

$$\frac{o_d}{o} = \frac{f_d}{f_d + o - x}\left(\frac{o - x}{o}\right) \tag{6}$$

Equations 5 and 6 give the changes, respectively, in the magnification ratio at the sensor and the minimum working distance. Each of these changes will affect the illuminance at the sensor. By the inverse square law for radiation, the reduced working distance increases the illuminance by the factor $(o/o_d)^2$. On the other hand, the increased magnification at the sensor distributes the captured light over a larger area and therefore decreases the illuminance by the factor $(m/m_d)^2$. These two competing effects combine to produce the following ratio of the illuminances *with* the close-up lens (I_d) and *without* the close-up lens (I) :

$$\frac{I_d}{I} = \left(\frac{o}{o_d}\right)^2 \left(\frac{m}{m_d}\right)^2 = \left(\frac{o}{o - x}\right)^2 \tag{7}$$

This ratio is therefore always greater than one, becoming identically equal to one when x=0. In other words, the use of a close-up lens to increase magnification at closest focus will actually **increase** the light intensity at the sensor. For values of x that are small in comparison with o, there is almost no change in illuminance.

Extension Tubes

Figure 3 shows the same lens as in Figure 1, but mounted on an extension tube of length D. The distance between the lens and the image plane is now "i+D" and the minimum working distance is now o_T.

For this configuration, the thin-lens formula yields

$$\frac{1}{o_T} + \frac{1}{D + i} = \frac{1}{f} \tag{8}$$

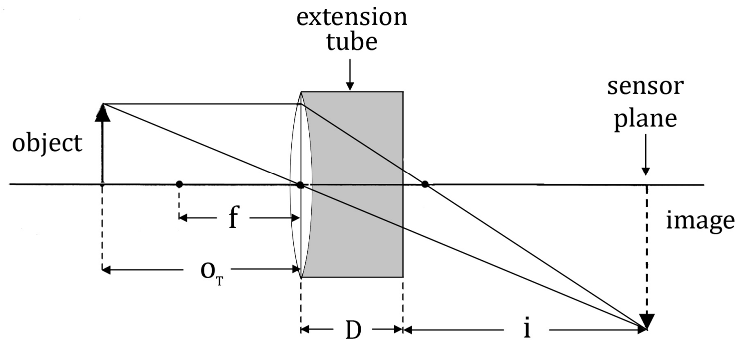

Figure 3

By analogy with Equation 2, the new magnification ratio is

$$m_T = \frac{f}{o_{T-f}} = \frac{D + i - f}{f} \tag{9}$$

Combining Equations 2 and 9, we get

$$\frac{m_T}{m} = \frac{D + i - f}{i - f} \tag{10}$$

Suppose that without the extension tube the magnification ratio is 1. From Equation 2, m= 1 corresponds to i= 2f. According to Equation 10, the addition of an extension tube with D=f will then yield m_T=2. The tube will **double** the magnification ratio.

Equations 1 and 8 produce

$$\frac{o_T}{o} = \frac{i - f}{i - f\left(\frac{i}{D + i}\right)} \tag{11}$$

Since this ratio is always less than one, the minimum working distance with the tube is always smaller than without the tube. Substituting i=2f and D=f shows that a doubling of the magnification ratio is achieved by reducing the lens-to-object distance by a factor of $\frac{3}{4}$.

Equations 10 and 11 then yield

$$\frac{o_T}{o} = \frac{m}{m_T}\left(1 + \frac{D}{i}\right) \tag{12}$$

As in Equation 7, the ratio of the illuminances *with* the extension tube (I_T) and *without* the extension tube (I) is

$$\frac{I_T}{I} = \left(\frac{o}{o_T}\right)^2 \left(\frac{m}{m_T}\right)^2 \tag{13}$$

Substituting from Equation 12 into Equation 13, we finally get

$$\frac{I_T}{I} = \left(\frac{1}{1 + \frac{D}{i}}\right)^2 \tag{14}$$

This ratio is always less than one if $D \neq 0$. In other words, the use of an extension tube to increase magnification will **decrease** the illuminance at the sensor. This is because the loss in illuminance due to the increased magnification outweighs the gain in illuminance due to the reduced working distance. For a 1:1 lens, the loss in exposure, in f-stops, is

$$\text{f/stop loss} = \frac{1}{2}\left(1 + \frac{D}{2f}\right)^2 \tag{15}$$

For example, for D=f, D=2f, and D=3f, the light losses are approximately 1, 2, and 3 f-stops, respectively.

Teleconverters

A teleconverter is a supplementary diverging lens that is inserted between a primary lens and the camera body in order to magnify the image at the sensor plane. In effect, it produces images that you would achieve with a primary lens having a longer focal length, with one major difference --- the addition of a teleconverter does not change the working distance relative to the primary lens.

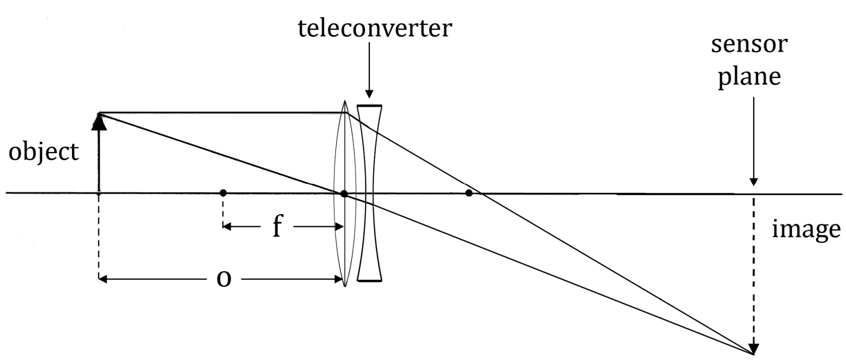

Figure 4

Figure 4 shows the same lens as in Figure 1, set to closest focus and with a teleconverter behind it. The teleconverter does two things --- it extends the primary lens forward to provide more distance between the lens and the sensor plane, and it refracts the rays leaving the primary lens so that they produce a magnified image at the sensor plane.

A teleconverter is designed to produce a particular magnification ratio, m_{TC} . When used with a primary lens that is producing a magnification ratio of m=1, the teleconverter spreads the light entering it over an area that is larger by the factor $(m_{TC})^2$, thereby decreasing the illuminace at the sensor plane by the same factor. However, unlike the cases with the close-up lens and extension tube, there is no increase in illuminance because the working distance does not decrease. Hence, at the sensor plane, the ratio of the illuminances *with* the teleconverter (I_{TC}) and *without* the teleconverter (I) is

$$\frac{I_{TC}}{I} = \left(\frac{1}{m_{TC}}\right)^2 \tag{16}$$

The loss in exposure, in f-stops, is

$$\text{f/stop loss} = \frac{1}{2}(m_{TC})^2 \tag{17}$$

For example, a 2x teleconverter (m_{TC}=2.0)cause a light loss of 2 stops.

The following table is based on the above results.

Light losses for a 1:1 primary lens with various supermacro tools

Supermacro Tool	Magnification ratio	loss in f-stops
Close-up lens	2	none
1 focal-length extension tube	2	1
1.4x teleconverter	1.4	1
1.7x teleconverter	1.7	1.5
2.0x teleconverter	2	2