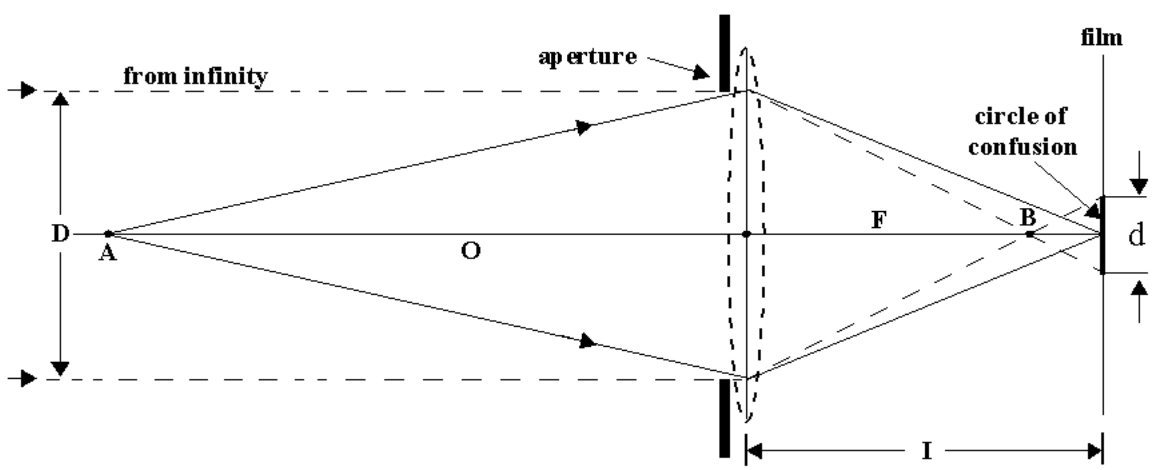
The Hyperfocal Distance

Consider the arrangement shown, consisting of a perfect converging lens, an aperture of diameter D, a point object A on the lens axis, and film. When the image of A is perfectly focussed at the film plane, some of the scene behind A and in front of A is also in focus.

Suppose that A is very far from the lens (at 'infinity'). By definition, light rays from A will be focused at the point B on the lens' secondary focal plane, which is a distance F (the focal length) behind the center of the lens. If the film plane is positioned to contain point B, the film will record a perfectly focused image of A. Now bring A closer, to a finite distance O from the lens, but still far enough away so that O is much larger than F. The image recedes to a distance 'I' behind the lens, as shown, in accordance with the Gaussian (lens-maker's) formula

$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F} \qquad (1)$$

and the film plane must move with it to record a sharp image. With object A at this new position, any light rays coming from other objects behind it are focused somewhere between the film plane and B, with the most out-of-focus rays being those coming from the most distant objects.



In particular, rays coming from infinity meet at B but continue on to form a **circle of confusion** at the film plane. By moving A back and forth, and moving the film accordingly to maintain a sharp image, the circle of confusion can be made larger or smaller. The bigger the circle the more out-of-focus will be the rays coming from objects at infinity. However, as long as the film cannot resolve details smaller than the circle of confusion, then as far as the film is concerned the images of such objects are **still in focus**. Moreover, since objects at infinity are the "worst-focused", then **any** objects beyond A will also be in focus. This will occur, for example, if the diameter "d" of the circle of confusion is smaller than the minimum grain separation of the film emulsion. So, what position of A does this correspond to?

From the congruent triangles formed by the rays passing through B, it is clear that the circle of confusion will have diameter d if

$$\frac{D}{F} = \frac{d}{I - F} \qquad (2)$$

If we solve for the image distance (I) from Equation (1), substitute the result into Equation (2), and solve for the object distance O, the result is the **hyperfocal distance,** H, given by the formula

$$H = \frac{F^2}{df}(1 + \frac{d}{D}) \qquad (3)$$

where we have put f = F/D, which by definition is the **f-stop** for the lens/aperture combination. In practice, d/D is much smaller than one, so the formula for the hyperfocal distance simplifies to the usual

$$H = \frac{F^2}{df} \qquad (4)$$

Thus, with point A a distance H in front of the lens and its image in perfect focus on the film, the images of all objects beyond A will appear to be equally in focus. Using similar geometrical considerations, one can also show that with A at the hyperfocal distance all objects from H/2 to H in front of the lens will **also** be in focus (see the article entitled “Field of Focus”). Hence, the depth of field extends from H/2 to infinity.

REFERENCE:

Young, M., "Optics and Lasers", 3rd ed., (Springer-Verlag, Berlin, Heidelberg 1986), chapter 2