Underwater Spectacles

When you open your eyes underwater the corneas of your eyes are in direct contact with water, instead of with air. Because the index of refraction of water is not very different from the index of refraction of your cornea, the cornea becomes much less effective in helping to focus light rays on the retina. In fact, the cornea loses about 42 diopters of refractive power.

For most people having difficulty focussing in air, the solution is to prescribe glasses that will correct their vision problem. In principle, if your vision problem happens to be not being able to see underwater, you should be able to correct it with an appropriate pair of glasses. Let’s see how we can accomplish that.

To restore the cornea’s lost refractive power you need a converging lens with an in-water power of +42 diopters. Since refractive power is the inverse of the focal length (in meters), the lens’ focal length must be 24 mm. If we want "normal-looking" glasses let’s assume that the lens is bi-convex and symmetric with spherical surfaces having radii of curvature of, say, 42 mm (see below).

So, we have designed everything but the lens’ index of refraction. What must it be?

For a bi-convex symmetric lens having radii of curvature "r", index of refraction N, and immersed in a medium of refractive index "n", the lens’ power is

$$P = \left(\frac{N - n}{n} \right) \frac{2}{r}$$

Requiring P = 42, n = 1.33 (water), and r = 42 mm, we find that the lens’ index of refraction will have to be

$$N = 2.5$$

Clearly the lenses cannot be made of ordinary crown glass, which has an index of refraction in the range 1.5-1.7. Yes, the glasses can be built, but they will be rather expensive --- **the lenses will have to be made of diamond**!

If lenses made of diamond are a bit out of your league, how about lenses made of **air**? In a 1924 paper (see the reference), Dr. J. A. Valentine proposed that lenses be made of a symmetric bi-concave sealed glass compartment containing air (see below).

Such a lens would have no refractive power in air, but in water it would act as a converging lens. In fact, the formula for its underwater refractive power is identical to the one given above, but with a radius of curvature that is negative instead of positive. In other words, the power of this lens is

$$P = \left(\frac{n - N}{n}\right)\frac{2}{r}$$

In this case the indices of refraction are fixed at the values n = 1.33 (water) and N = 1.00 (air), but the radius of curvature "r" is adjustable. As before, we want P = 42, which implies that the radius of curvature must be 14 mm.

On the other hand, you could just go out and buy a mask.

Reference:

J. A. Valentine, "A Pair of Diving Spectacles", Proc. R. Soc. Med. **17**, Sect. Ophthal.17, 1924.