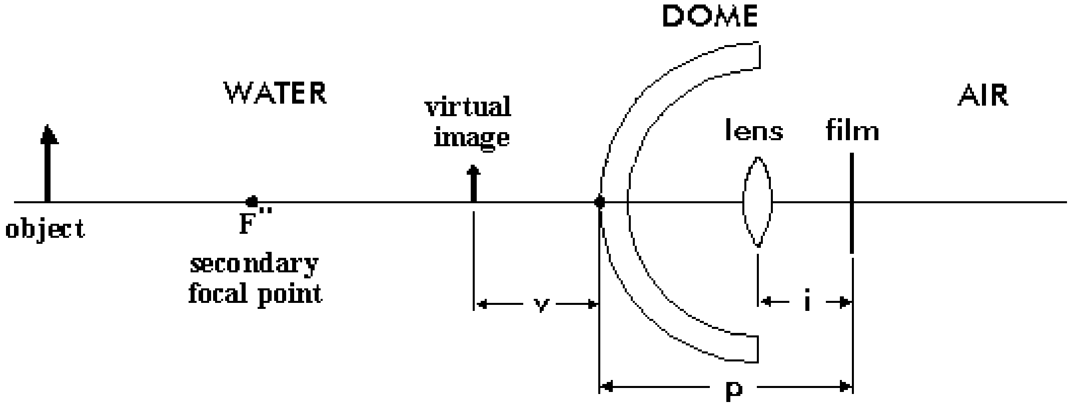
Domes, Diopters, and Depth of Field

Wide angle underwater photography using a housed SLR requires the use of a hemispherical dome port. Unlike a flat water-air interface, which drastically reduces a lens' field of view, a spherical interface is capable of fully restoring the lens' in-air capture angle.

Although this solves one problem it also introduces another: you must now focus on something that isn't really there. When light rays from a point object cross the interface from water to air they are bent away from the optical axis (diverge) and seem to be originating from a "virtual" image point close to the front of the dome. In fact, all of "real object" space in front of the dome is compressed into a "virtual image" space extending from the front of the dome to less than 3 dome radii away. The virtual images within (at least some of) this range must be converted into real images at the film plane. If the virtual images are too close for the camera's lens ("primary lens") to focus on, you will need to attach a supplementary closeup lens.

The addition of an appropriate corrective lens lets you focus where you need to, but it also raises several questions about potential degradation of the image. One of the questions is: "What happens to the depth of field?" More precisely: "If you take a picture of an object using just the camera lens, then add a closeup lens and do whatever you have to do to get an identically-sized picture of the same object, which photo will show greater depth of field?

To answer this question let's assume that the primary and supplementary lenses are very thin, single-element, converging lenses with respective focal lengths f and f_d. This will simplify the analysis without invalidating any conclusions. The figure below shows a dome port with secondary focal point F", a moveable lens near the dome's center of curvature, a film plane at a fixed distance behind the dome, a small object, and its corresponding virtual image produced by the dome.



Note that an object point on the optical axis but at infinity will produce a virtual image point at F”, while closer object points will fall somewhere between F” and the front of the dome.

Assume first that the lens in the diagram is the primary lens and that the virtual image is focussed at the film plane (if the object is small enough the virtual image’s curvature can be neglected). We have the following relationship:

$$\frac{1}{v + p - i} + \frac{1}{i} = \frac{1}{f} \tag{1}$$

Note that **i** is an implicit function of **v**. The size of the real image at the film plane divided by the size of the virtual image is the lens’ magnification ratio, $m_{lens}(v)$,

$$m_{lens}(v) = \frac{i}{f} - 1 \tag{2}$$

where its dependence on the virtual image’s position is explicitly shown. The size of the virtual image divided by the size of the object is the dome’s magnification ratio, m_{dome} (v), which depends on the object’s (and hence the virtual image’s) position. The total magnification is therefore

$$m(v) = m_{dome}(v)\, m_{lens}(v) \tag{3}$$

Suppose that you now add the supplementary lens to the primary lens. Because they are both thin lenses, their combination can be treated as a single thin lens with the focal length

$$f_{eff} = \frac{f_d}{f + f_d} f \tag{4}$$

Note that f_{eff} is **smaller** than f. With nothing else changed the real image produced by this effective lens will therefore be focussed **in front of the film plane**, so the lens must be moved towards the film until, at some new image distance $(<i)$, it will again be focussed at the film plane. Since the image distance is smaller and the lens-to virtual-image distance is greater, and the ratio of these two distances is another expression for the lens magnification ratio, the size of the image will now be **smaller.** Our objective, of course, is to end up with an image that is **identical in size** to the one produced by the camera lens alone, so that we can fairly compare the depths of field in the two scenarios. Clearly, the way to achieve that objective is to **move the virtual image towards the film plane** while readjusting the position of the lens to maintain focus. (Note: the virtual image can be moved only by changing the dome-to-object distance). For an appropriate value of the virtual image’s position, say v_1, and a corresponding real image distance i_1, we will therefore satisfy the condition

$$m(v_1) = m(v) \tag{5}$$

or

$$m_{dome}(v_1) \left(\frac{i_1}{f_{eff}} - 1 \right) = m_{dome}(v) \left(\frac{i}{f} - 1 \right) \tag{6}$$

A method for determining the dome's magnification ratio has been given in the article entitled "Optics of Dome Ports". Equation (6) can be solved for v_1 and i_1 as follows:

- Guess a value for v_1 .
- Calculate $m_{dome}(v_1)$ using the formula from "Optics of Dome Ports".
- Calculate i_1 using the equation

$$\frac{1}{v_1 + p - i_1} + \frac{1}{i_1} = \frac{1}{f_{eff}} \tag{7}$$

- Using these values of v_1 and i_1, evaluate the left hand side of Equation (6) and compare it to the right hand side. If they are different, try a new guess for v1.
- Repeat the cycle until the equation is satisfied.

Having solved Equation (6) you now have, for the combined lens, the position v_1 of the virtual image (and the position i_1 of its real image) that gives the same object magnification produced by the camera lens alone when the virtual object is at v. The final images on the film are exactly the same size, but which one has better depth of field? In the absence of any other image degradation effects, we can answer this question as follows:

- For each lens (primary, primary + supplementary) compute the near and far focus limits in **virtual image space** when the lens is focussed at the appropriate virtual image position (v or v_1).
- For each lens, determine the object positions corresponding to these near and far focus limits. The distance between the near and far object positions is the depth of field in object space.

Formulas for the near and far limits of the depth of focus can be found in the article entitled "Field of Focus".

The results show that, in general, there is a **reduction in depth of field** when a supplementary closeup lens is attached to the camera lens. An increase in the focal length of the camera lens or in the power of the closeup lens will produce a larger reduction. Some results for a quarter-inch thick 3-inch dome are shown below.

Loss of depth of field

Camera lens focal length	**+3 diopter closeup lens**	**+4 diopter closeup lens**
18 mm	5%	7%
28 mm	8%	11%
35 mm	11%	14%

This loss of depth of field is a consequence of the shift of the virtual image towards the dome. At the “infinity” end of the virtual image space there is a high degree of compression --- widely separated objects have their virtual images squeezed closely together. In this region a short distance in virtual image space corresponds to a large one in object space. At the other end of the virtual image space (the front of the dome) the compression is not as great, and the same short distance spans less of object space.

When the finite separation between the primary and supplementary lenses are accounted for, the depth of field losses are slightly reduced.