

# Closeup Lenses and Magnification

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When you attach a closeup lens to your camera's lens there will be a change in the size of the image at the film plane. The magnitude (and sign) of the change depend on the lens properties and the distance to the subject.

For a thin lens we have the usual relationship:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f} \quad (1)$$

where  $o$  and  $i$  are the object and image distances respectively (from the centre of the lens), and  $f$  is the lens' focal length. The magnification ratio is (using Eq.(1))

$$m = \frac{i}{o} = \frac{f}{o-f} = \frac{i}{f} - 1 \quad (2)$$

Also, from Eq.(1),

$$o = \frac{f}{i-f} i \quad (3)$$

Suppose that you now add a closeup lens (a "diopter") of focal length  $f_d$ . Assuming that both are thin lenses, their combination can be treated as a single thin lens with the focal length

$$f_{\text{eff}} = \frac{f_d}{f + f_d} f \quad (4)$$

Note that  $f_{\text{eff}}$  is **smaller** than either  $f$  or  $f_d$ .

If you leave the image-to-lens distance "i" fixed at its previous value, the only way to re-focus the image at the film plane is by adjusting the object distance. The required object distance is given by Eq.(1) with  $f$  replaced by  $f_{\text{eff}}$ , i.e.

$$o_{\text{new}} = \frac{f_{\text{eff}}}{i - f_{\text{eff}}} i \quad (5)$$

But we know from Eqs.(3) and (4) that  $i > f > f_{\text{eff}}$ . Comparing Eq.(5) with Eq.(3) we therefore have

$$o_{\text{new}} < o \quad (6)$$

The lens-to-object distance has been **reduced**. The new magnification ratio,  $m_1$ , is (from Eq.(2))

$$m_1 = \frac{i}{f_{\text{eff}}} - 1 = \frac{(m+1)f}{f_{\text{eff}}} - 1 = m + \left\{ (m+1) \left( \frac{f}{f_{\text{eff}}} - 1 \right) \right\} \quad (7)$$

where  $m$  is the magnification ratio without the diopter. Since  $f > f_{\text{eff}}$  it is clear that the quantity within the curly brackets is positive, and

$$m_1 > m \quad (8)$$

**Conclusion: If you add a closeup lens and change the lens-to-object distance to restore focus, the magnification ratio will increase.**

What if, on the other hand, the object distance had been fixed while the image distance was adjusted to restore focus? From Eq.(2),

$$o = f \left( 1 + \frac{1}{m} \right) \quad (9)$$

In this case the new magnification ratio,  $m_2$ , becomes (combining Eq.(2) and Eq.(9))

$$m_2 = \frac{f_{\text{eff}}}{f \left( 1 + \frac{1}{m} \right) - f_{\text{eff}}} = m + \left\{ m(m+1) \frac{\alpha - 1}{1 + m(1 - \alpha)} \right\} \quad (10)$$

where  $\alpha = f_{\text{eff}}/f$ . Since  $\alpha < 1$  the quantity inside the curly brackets is negative. Hence

$$m_2 < m \quad (11)$$

**Conclusion: If you add a closeup lens and do not change the lens-to-object distance to restore focus, the magnification ratio will decrease.**

Let's apply the result of Eq.(7) to a 60 mm lens and a 105 mm lens, and a +1 diopter closeup lens (Note: a closeup lens having a power of +4 diopters in air will have a power of +1 diopter in water). The table below summarizes the results. Refer to Eqs.(2) and (7) for notation.

## MAGNIFICATION RATIOS

	60 mm lens		105 mm lens	
m	$m_1$	$m_1/m$	$m_1$	$m_1/m$
.050	.113	2.26	.160	3.20
.067	.131	1.96	.179	2.67
.100	.166	1.66	.216	2.16
.125	.193	1.54	.243	1.94
.500	.590	1.18	.658	1.32
1.000	1.12	1.12	1.21	1.21

[m = magnification ratio for primary lens alone

$m_1$  = magnification ratio for primary lens plus a +1 diopter lens]

The foregoing was based on the “thin lens” approximation. Since “real” camera lenses are not thin the absolute magnification ratios given by the formulae should not be considered exact, but only good approximations. Relative magnification ratios (e.g.  $m_1/m$ ) and changes in magnification ratios should be even better.